Paramagnetic phases of Kagome lattice quantum Ising models

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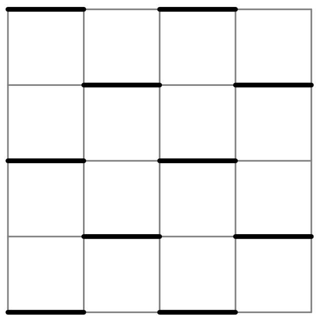
Overview

- Introduction
  - Quantum Ising models
  - Motivation
- Transverse field Ising model on the Kagome lattice
  - Dynamics of individual spins
  - Disordered phase for all strengths of transverse field
- XXZ model on the Kagome lattice
  - Dynamics of frustrated bonds
  - Disordered, spin liquid and valence-bond ordered phases

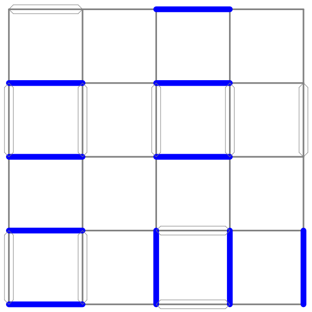
Quantum Paramagnetic Phases

- Singlet valence-bond: $|\bullet\!\!-\!\!\bullet\rangle \sim \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$

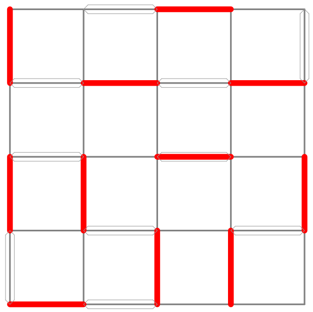
staggered VBC



plaquette VBC

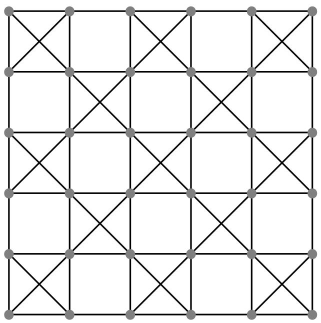


spin liquid (RVB)

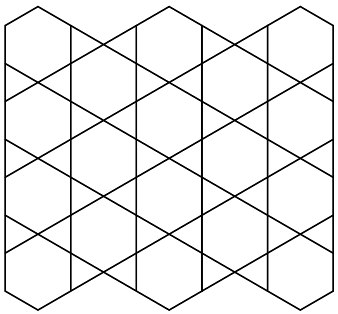


- Promising spin systems:

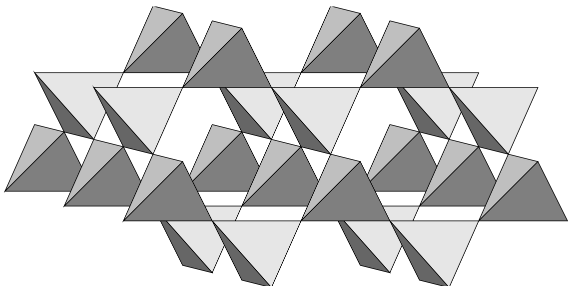
Checkerboard



Kagome



Pyrochlore



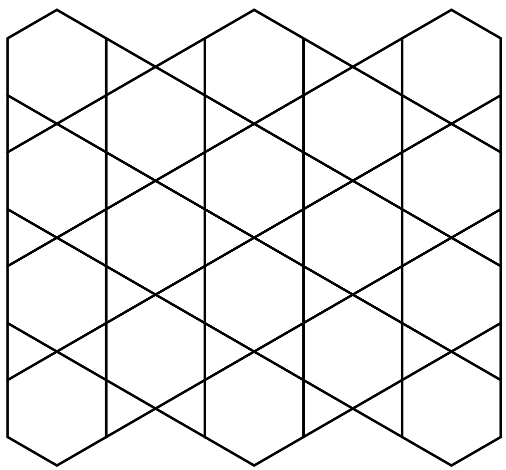
Models

- Transverse field quantum Ising model (TFIM)

$$H = J_z \sum_{\langle ij \rangle} S_i^z S_j^z - \Gamma \sum_i S_i^x \qquad , \qquad \Gamma \ll J_z$$

- XXZ model: total Ising spin conserved

$$H = J_z \sum_{\langle ij \rangle} S_i^z S_j^z + J_\perp \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) \qquad , \qquad |J_\perp| \ll J_z$$



- $S = \frac{1}{2}$
- Nearest-neighbor Ising interaction
- Further-neighbor and multiple-spin exchange dynamics
- What phases are possible?

Motivation for Kagome Ising Models

- Kagome Heisenberg a.f.
  - Seemingly gapless modes in absence of continuous symmetry breaking
  - Easy-axis anisotropy $\Longrightarrow$ XXZ model

Ch. Waldtmann
H.-U. Everts
B. Bernu
P. Sindzingre
C. Lhuillier
P. Lecheminant
L. Pierre

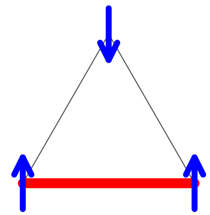
- Kagome Ising a.f. in weak transverse fields
  - Disordered ground-state

R. Moessner
S. L. Sondhi

- Search for unconventional quantum phases
- Conditions in which various phases occur
  - Kagome phases: disordered, spin liquid, VBC

U(1) Gauge Theory: Spin Picture

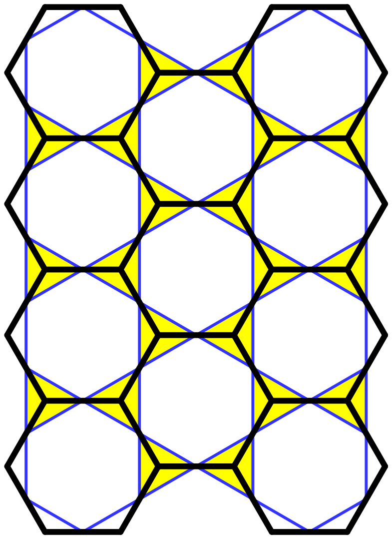
$$H_z = J \sum_{\langle ij \rangle} S_i^z S_j^z = \frac{J}{2} \sum_{\triangle} \Big(\sum_{i \in \triangle} S_i^z \Big)^2 + \text{const.}$$



- Minimum frustration $\Longrightarrow$ local constraint

$$(\forall \triangle_p) \quad s_p^z = \sum_{q \in p} S_{\langle pq \rangle}^z \in \left\{ \pm \frac{1}{2} \right\}$$

- U(1) gauge theory on honeycomb lattice
  - Electric field vector E_{pq}
  - Charged boson n_p



Mapping	lattice		quantity		condition
Kagome	site i	triangle $\triangle_p$	S_i^z	s_p^z	min. frustration
honeycomb	bond $\langle pq \rangle$	site p	E_{pq}	n_p	Gauss' Law

Analysis

- 2D compact U(1) gauge theory with bosonic matter field
  $\Longrightarrow$ a non-topological disordered phase exists (in 2D)

1. Duality transformation
   $\Longrightarrow$ integer-valued gauge theory on triangular lattice
   study dynamics of *vortices*
2. Relax integer-constraints
   $\Longrightarrow$ sine-Gordon theory
3. Integrate out high-energy fields
   $\Longrightarrow$ effective XY model with 3-state anisotropy
4. Find continuum limit
   $\Longrightarrow$ field theory, explore the phase diagram

Phases: Transverse Field Ising Model

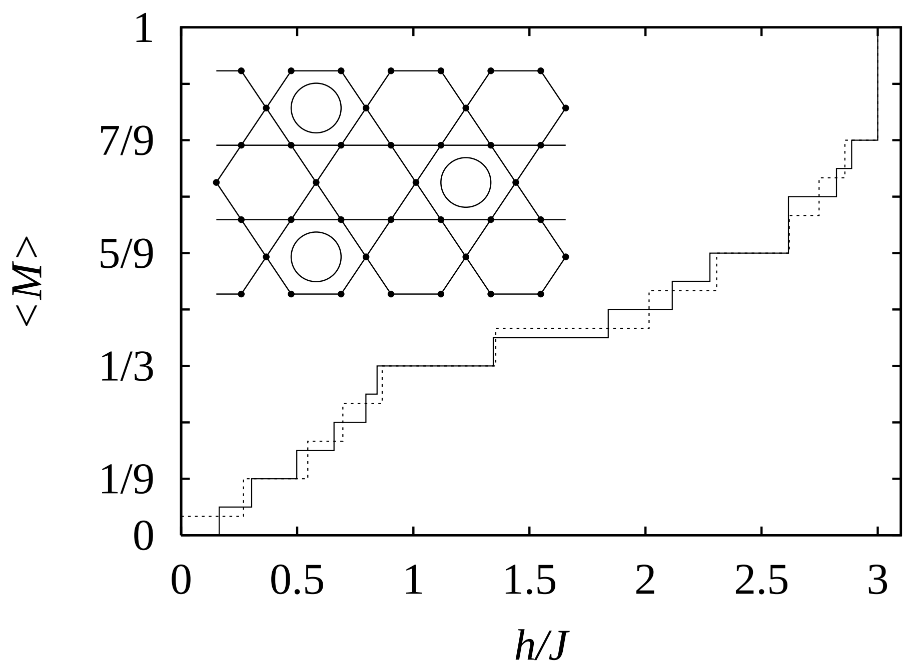
- Disordered non-topological phase (Higgs)

  agrees with Monte-Carlo (Moessner, Sondhi)

- Valence-bond ordered phase

  broken translational symmetry (3-fold degeneracy)

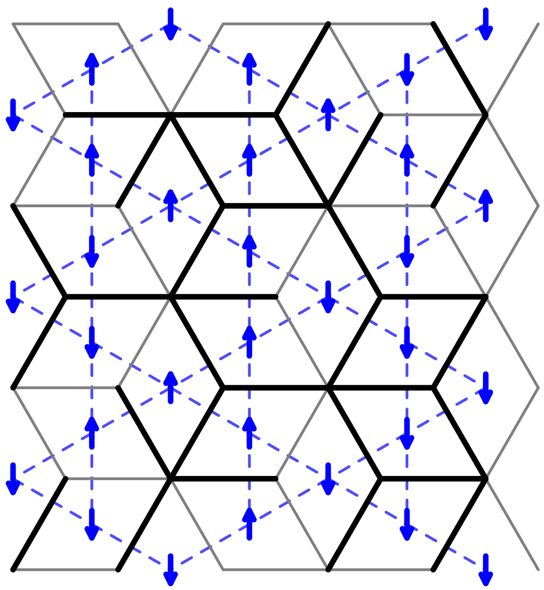
  broken global $\mathbb{Z}_2$ symmetry



Dominant hexagon ring exchange. . .

Ordered phase is perhaps related to the plateau in magnetization curve

U(1) Gauge Theory: Bond Picture



- Frustrated bond $|\uparrow\uparrow\rangle \Leftrightarrow$ dimer
- Minimal frustration:
  - one dimer per Kagome triangle
  - many degenerate "ground-states"

- Quantum fluctuations lift degeneracy
- How: Quantum dimer model on the *dice* lattice
- Order-by-disorder?
  - entropically selected ordered state?

Outline of Calculations

- Dice lattice is bipartite
  - quantum dimer model $\Leftrightarrow$ compact U(1) gauge theory
- Dimers are soft-core
  - bosonic matter field in the gauge theory
  - distinguishes Kagome from other 2D frustrated systems
  - disordered phases are possible
- Duality transformation
  - lattice field theory (“height” model)
- Exploring the phase diagram
  - field-theoretical methods: “extended mean-field”

Extended Mean-Field Method

- Mean-field state is determined by minimum of energy and maximum of entropy $\Longrightarrow$ minimize "free energy"
- Take into account effects of quantum fluctuations $\Longrightarrow$ seek order-by-disorder

Formalism:

- Consider a path-integral with action $S(\Phi)$
- Calculate free energy $F(\Phi_0)$ of a *microstate* Φ_0:

$$e^{-F(\Phi_0)} = \int \mathcal{D}\Phi \;\; e^{-S(\Phi) - m^2 \int \mathrm{d}^d \boldsymbol{r} \left(\Phi(\boldsymbol{r}) - \Phi_0(\boldsymbol{r})\right)^2}$$

- Find the microstates Φ_0 that minimize $F(\Phi_0)$

Transverse Field Ising Model

- Dimer flips consistent with minimal frustration:

- Result: “disorder-by-disorder”
  - entropically selected states are macroscopically numerous and generally disordered

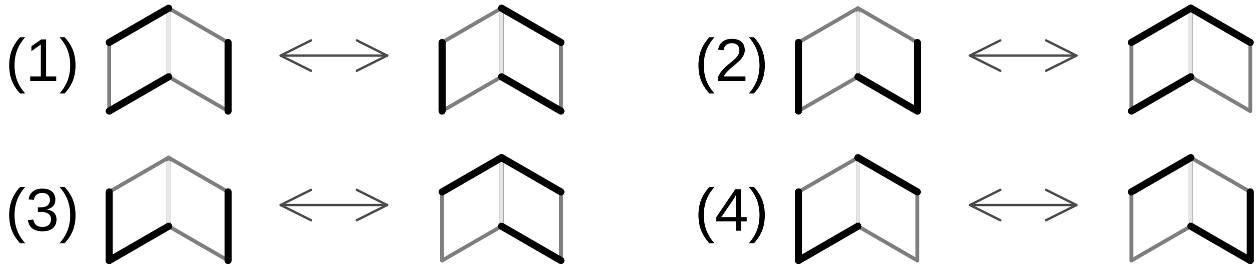
- No phase transitions as $\Gamma \to \infty$
- Agrees with Monte-Carlo:

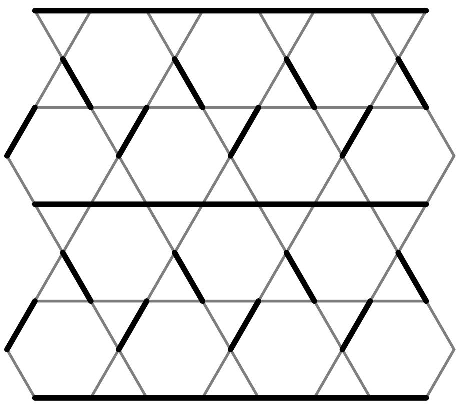
  R. Moessner, S. L. Sondhi; Phys. Rev. B **63**, 224401 (2001)
- Different theory, same result:

  U(1) gauge theory on the honeycomb lattice

XXZ and Spin-Conserving Models

- More complicated dimer dynamics:





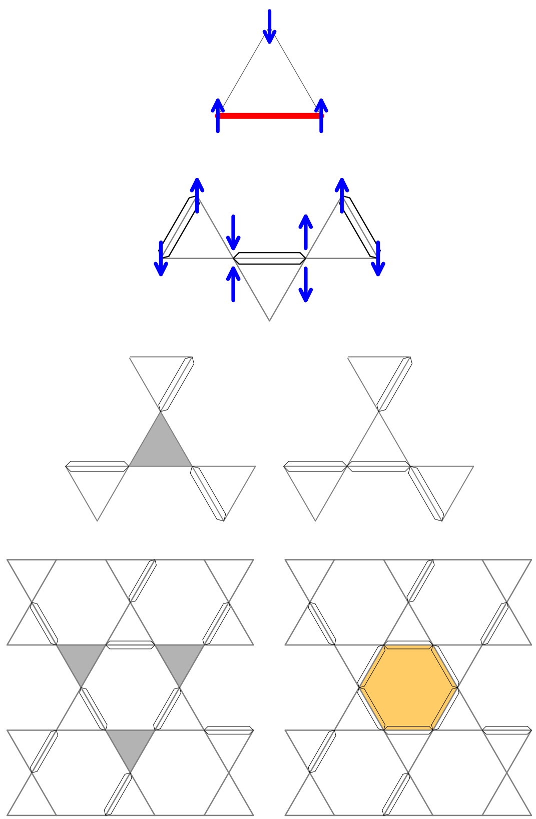
Result: two possible phases

- valence-bond crystal (XXZ)
- spin liquid (multiple-spin exchange. . .)

Any phase with no broken lattice symmetries and conserved total Ising moment has topological order.

XXZ Variational States

$$H = J_z \sum_{\langle ij \rangle} S_i^z S_j^z \pm J_\perp \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) \qquad , \qquad 0 < J_\perp \ll J_z$$



- J_z: Minimize Ising frustration
- $J_\perp$: Maximize the number of valence-bonds: $|\Leftrightarrow\rangle \sim |\uparrow\downarrow\rangle \mp |\downarrow\uparrow\rangle$
- J_z: Avoid frustration on the *defect* triangles: $|\Leftrightarrow\Leftrightarrow\Leftrightarrow\rangle \sim |\uparrow\downarrow\uparrow\downarrow\rangle \mp |\downarrow\uparrow\downarrow\uparrow\rangle$
- $J_\perp$: Minimize the cost of *defect* triangles by maximizing the number of *perfect* hexagons

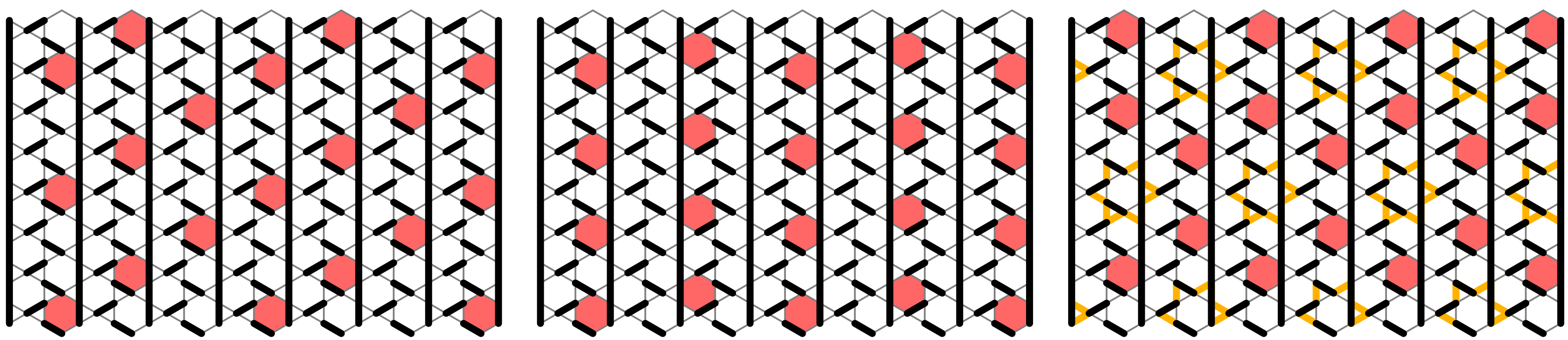
XXZ Variational States

Visual overlap between the entropically selected state and variational states:

$$|\ \rangle \sim |\uparrow\uparrow\rangle \vee |\downarrow\downarrow\rangle$$

$$|\ \rangle \sim |\uparrow\downarrow\rangle \mp |\downarrow\uparrow\rangle$$

$$|\ \rangle \sim |\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\rangle \mp |\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\rangle$$

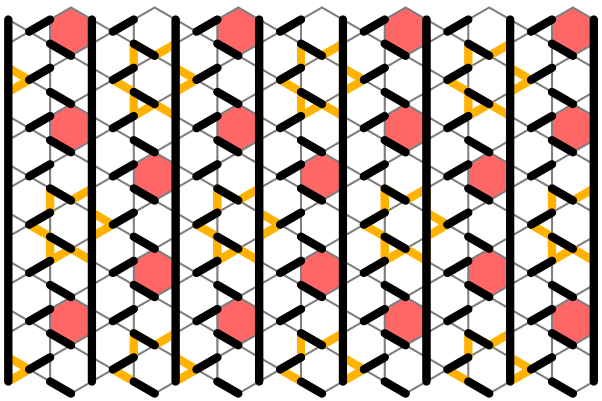


Valence-bond order proposed for the Heisenberg model

P. Nikolic, T. Senthil; Phys. Rev. B **68**, 214415 (2003)

Conclusions: Kagome Ising Phases

type of dominant Ising dynamics:	simple short-ranged	multiple-spin and ring exchange. . .
does not conserve $\sum_i S_i^z$	disordered (TFIM)	disordered
conserves $\sum_i S_i^z$	valence-bond crystal (XXZ)	spin liquid
hexagon ring-exchange	magnetized valence-bond crystal	



Valence-bond order proposed for the Heisenberg model

Found in the effective $\mathbb{Z}_2$ gauge theory of the “gapless” singlets